Radiative tail from the quasielastic peak in deep inelastic scattering of polarized leptons off polarized ^{3}He

I.Akushevich[1,2], D.Ryckbosch[3], N.Shumeiko[1], A.Tolkachev[1]

[1] National Center of Particle and High Energy Physics, Minsk Belarus
[2] NC Central University NC and TJNAF VA USA
[3] Gent University, Belgium

Abstract. The contribution of the radiative tail from the quasielastic peak to low order radiative correction to deep inelastic scattering of polarized leptons by polarized ^{3}He was calculated within the sum rules formalism and y-scaling hypothesis. Numerical analysis was carried out under the conditions of HERMES experiment.

PACS numbers: 13.40.K, 13.60.-r, 13.88.+e

Submitted to: *J. Phys. G: Nucl. Part. Phys.*

1. Introduction

There is great interest in the investigation of the spin structure of nucleons in modern high energy physics. Experiments on deep inelastic scattering (DIS) of polarized leptons by polarized light nuclei are among the most powerful sources to obtain such information. The interpretation of results of these experiments requires the correct treatment of radiative corrections (RC), to minimize their contribution to the systematic errors.

It is well known that there are three channels for scattering of a virtual photon by a nucleus, depending on the energy transfer $\nu = E_1 - E_2$, with $E_1(E_2)$ the initial (scattered) lepton energy: elastic, quasielastic and inelastic. Both ν and the momentum transfer (squared) $Q^2 = -q^2$ are fixed by kinematical conditions on the Born level and, consequently, also the scattering channel is fixed. However, on the RC level the uncertainty in the energy of the radiated photon makes Q^2 and ν indefinite and as a result each of the three channels contributes to the cross section. Whereas contributions from the elastic and inelastic tails are well studied, the contribution from the quasielastic tail was not thoroughly investigated. Up to now there are only approximate results, for a few target nuclei, and based on phenomenological models[1, 2] using the Mo and Tsai [3] formalism. For ^{3}He no results were published so far. Only the general framework exists for the case of polarized reactions [4]. In this paper the cross section and the polarization asymmetry of DIS of polarized charged leptons from polarized targets (H, D, ^{3}He) have been investigated both on the Born level and taking into account RC. The purpose of the present paper is to develop the methods that are necessary to correctly treat the radiative tail from the quasielastic peak. This is then applied to the specific case of a ^{3}He target, using approximate (due to the lack of experimental data), but model independent approaches. Numerical results are presented relevant for the kinematics of the HERMES experiment.

The paper is organized as follows. In section 2 we derive the basic formulae and consider y-scaling. Section 3 is devoted to the implementation of sum rules approach to the calculation of RC. Finally, in section 4 we present the numerical analysis.

2. Radiative tail from the quasielastic peak

The model independent RC of the lowest order can be written as a sum of bremsstrahlung and loop effects:

$$\sigma = \sigma_{in} + \sigma_{el} + \sigma_q + \sigma_v. \tag{1}$$

Each σ denotes a double differential cross section $d^2\sigma/dxdy$, with x, y the usual scaling variables; $\sigma_{in,el,q}$ are the contributions of the radiative inelastic, elastic and quasielastic (QRT) tails. The term σ_v contains the contribution of virtual photon radiation and vacuum polarization effects.

The exact contribution of the QRT to total RC of the lowest order in the case of DIS of polarized leptons off polarized nuclei is given by

$$\sigma_q = -\frac{\alpha^3 S_x S}{\pi \lambda_s} \int \frac{d^3k}{k_0} \frac{1}{(q-k)^4} \mathcal{L}_{\mu\nu} W_{\mu\nu}(p, q-k). \tag{2}$$

Invariants are defined as usual:

$$\begin{gathered} S = 2k_1 p,\ X = 2k_2 p = (1-y)S,\ Q^2 = -(k_1 - k_2)^2 = xyS, \\ S_x = S - X,\ \lambda_s = S^2 - 4m^2M^2, \end{gathered} \tag{3}$$

where $k_1(k_2), m$ are the initial (final) lepton momentum and its mass respectively, k is the momentum of the radiated photon and p, M are momentum and the mass of the initial nucleon.

The leptonic tensor we use in (2) is standard for bremsstrahlung processes and includes spin averaged and spin dependent parts [5]. For the hadronic tensor we have

$$\begin{aligned} W_{\mu\nu}(p,q) =& -(g_{\mu\nu} + \frac{q_\mu q_\nu}{Q^2})\mathcal{F}_1 + \frac{1}{M^2}(p_\mu + \frac{pq}{Q^2}q_\mu)(p_\nu + \frac{pq}{Q^2}q_\nu)\mathcal{F}_2 + \\ &+ \frac{i}{M}\epsilon_{\mu\nu\alpha\beta}q_\alpha\eta_\beta\mathcal{F}_3 - \frac{i}{M^3}\epsilon_{\mu\nu\alpha\beta}q_\alpha p_\beta(q\eta)\mathcal{F}_4, \end{aligned} \tag{4}$$

where η is a target polarization vector. The quantities $\mathcal{F}_i$ are defined as some combinations of the nucleon structure functions. The dependence of the hadronic tensor on pq and the polarization of the beam (P_L) and target (P_N) is also included in the $\mathcal{F}_i$. Defining $\epsilon = M^2/pq$ we have for the various $\mathcal{F}_i$:

$$\begin{aligned} &\mathcal{F}_1 = F_1, \qquad \mathcal{F}_3 = P_N\,\epsilon\,(\,g_1 + g_2\,)\,, \\ &\mathcal{F}_2 = \epsilon F_2, \quad \mathcal{F}_4 = P_N\,\epsilon^2\ \ g_2. \end{aligned} \tag{5}$$

Definitions of SF $F_{1,2}$ and $g_{1,2}$ are the same as in ref. [6].

As was shown in [4] formula (2) leads to the following expression:

$$\sigma_q = -\frac{\alpha^3 y}{A} \int\limits_{\tau_{min}}^{\tau_{max}} d\tau \sum_{i=1}^{4}\sum_{j=1}^{k_i} \theta_{ij}(\tau) \int dR \frac{R^{j-2}}{\tilde{Q}^2} \mathcal{F}_i^q(R,\tau). \tag{6}$$

The variables R and τ are defined as

$$\begin{aligned} &R = 2pk, \qquad \tau = (\tilde{Q}^2 - Q^2)/R, \\ &\tilde{Q}^2 = -(k_1 - k - k_2)^2 = Q^2 + R\tau, \\ &\tau_{max,min} = \frac{S_x \pm \sqrt{S_x^2 + 4M^2Q^2}}{2M^2} \end{aligned} \tag{7}$$

and $k_i = (3, 3, 4, 5)$. The explicit form of the functions $\theta_{ij}(\tau)$ can be found in Appendix B of ref. [4].

The quantities $\mathcal{F}_i^q$ could be obtained in terms of quasielastic structure functions (so-called response functions, see Appendix A of [7] for explicit results), which are peaked around $\nu_q = Q^2/2M$. Due to the absence of sufficient experimental data this fact is

normally used as the basis of the peaking approximation: the response functions are estimated at the position of the peak, and a subsequent integration over the peak leads to results in terms of suppression factors $S_{E,M,EM}$ or (see discussion below) of sum rules for electron-nucleus scattering [8]:

$$\sigma_1^q = -\frac{\alpha^3 y}{A} \int\limits_{\tau_{min}}^{\tau_{max}} d\tau \sum_{i=1}^{4} \sum_{j=1}^{k_i} \theta_{ij}(\tau) \frac{2M^2 R_q^{j-2}}{(1+\tau)(Q^2+R_q\tau)^2} \mathcal{F}_i^q(R_q,\tau), \tag{8}$$

where $R_q = (S_x - Q^2)/(1+\tau)$.

In particular, for a ^{3}He target one finds:

$$\begin{aligned}
\mathcal{F}_1^q &= \eta(\mu_n^2 + 2\mu_p^2) S_M^u, \\
\mathcal{F}_2^q &= \frac{\eta(\mu_n^2 + 2\mu_p^2) S_M^u + (e_n^2 + 2e_p^2) S_E}{1+\eta}, \\
\mathcal{F}_3^q &= P_N 2(P_n e_n \mu_n + 2P_p e_p \mu_p) S_{EM}, \\
\mathcal{F}_4^q &= \frac{P_N}{4} \frac{(P_n e_n \mu_n + 2P_p e_p \mu_p) S_{EM} - (P_n \mu_n^2 + 2P_p \mu_p^2) S_M^p}{1+\eta}.
\end{aligned} \tag{9}$$

P_p and P_n are the effective proton and neutron polarization in ^{3}He and $e, \mu_{p,n}$ are the electric and magnetic formfactors for proton and neutron, $\eta = \tilde{Q}^2/4M^2$.

Explicit forms of the suppression factors (or functions $\mathcal{F}_i^q$) depend on a nuclear model for quasielastic scattering. Assuming the validity of y-scaling for quasielastic scattering [9] we have [10]

$$S_M^u = S_M^p = S_E = S_{EM} = F(\nu_q), \tag{10}$$

where $F(\nu_q)$ is a scaling function evaluated at the quasielastic peak. Neither a fit to experimental data nor a simple model for scaling function exist today. In the current version of the radiative correction code POLRAD, an extrapolation of a Fermi gas model [11, 12] is used which is really only applicable in heavier nuclei.

3. Quasielastic radiative tail and sum rules

Another possibility to obtain explicit forms for the suppression factors is to use the sum rules for electron-nucleus scattering [8, 13].

$$m_I^n = \int d\nu (\nu - \nu_q)^n R^I, \tag{11}$$

where R^I ($I = L, T, T', TL'$) are quasielastic response functions [8].

The functions $\mathcal{F}_i$ are linear combinations of these quasielastic response functions, which are supposed to have a form of a peak over transfer energy ν. This fact can be used for construction of some general expansion of coefficients in front of $R^{L,T,T',TL'}$ over $(\nu - \nu_q)$. Integration over $dR = 2Md\nu$ gives us just sum rules (11)

$$\int dR \frac{R^{j-2}}{\tilde{Q}^2} \mathcal{F}_i^q(R,\tau) = \int dR \frac{R^{j-2}}{\tilde{Q}^2} \sum_I C_i^I R^I = \qquad (12)$$
$$= \sum_{n,I} C_{in}^I \int d\nu (\nu - \nu_q)^n R^I = \sum_{n,I} C_{in}^I m_I^n$$

The sum rules are defined as a sum over final states of the cross section for the inclusive elastic and quasielastic scattering. Using the completeness property of final state nuclear wavefunctions the sum rules for electron-nucleus scattering are obtained as a sum of contributions of elastic and quasielastic scattering integrated over energy.

Since inclusive scattering is defined by two kinematical variables, sum rules correspond to the choice of external variables being some function of Q^2 and ν. In our case we have to keep τ as an external variable. However if to restrict the consideration to the lowest order of expansion (12) the difference between the sum rules due to the choice of the external variable is of the next neglected order, as it is always possible to recalculate it using ν taken at the peak.

For the calculation we use sum rules obtained in the paper [8], but subtracting an elastic contribution. The cross section for quasielastic scattering needed here is then obtained in terms of well defined model independent quantities (sum rules) and experimentally measured quantities (elastic formfactors):

$$\frac{d\sigma_q}{d\Omega} = \frac{d\sigma_{SR}}{d\Omega} - \frac{d\sigma_{el}}{d\Omega}, \qquad (13)$$

where Ω is lepton solid angle.

Using the explicit results for sum rules given in e.g. [8] and performing explicit subtraction of elastic contribution (13) the suppression factors can be obtained in the following form:

$$\begin{aligned}
S_M^u &= 1 - \frac{2\mu_p^2 T_q}{2\mu_p^2 + \mu_n^2} - \frac{\upsilon_{TA}}{\upsilon_T}\frac{\eta_A}{\eta}\frac{f}{f_A}\frac{4G_M^2}{2\mu_p^2 + \mu_n^2}, \\
S_E &= 1 + \frac{2(e_p^2 + 2e_p e_n)T_q}{2e_p^2 + e_n^2} - \frac{\rho_A}{\rho}\frac{f}{f_A}\frac{4G_E^2}{2e_p^2 + e_n^2}, \\
S_M^p &= 1 - \frac{\upsilon'_{TA}}{\upsilon'_T}\frac{\eta_A}{\eta}\frac{f}{f_A}\frac{4G_M^2}{\mu_p^2 P_p + \mu_n^2 P_n}, \\
S_{EM} &= 1 + \frac{2e_p\mu_n T_q}{e_p\mu_p P_p + e_n\mu_n P_n}\frac{Q_A'^2}{Q'^2}\frac{M}{M_A}\frac{q}{q_A}\frac{f}{f_A}\frac{4G_E G_M}{\mu_p e_p P_p + \mu_n e_n P_n},
\end{aligned} \qquad (14)$$

where we use the structure function T_q, which is the Fourier transform of the two-body density matrix (ref.[8]) and take for T_q results of ref.[14]. The last terms in righthandside of Eqs.(14) correspond to elastic scattering off ^{3}He and are obtained in terms of nuclear formfactors G_E and G_M. The other stem from the sum rule expressions (14a-d) of ref.[8] and depend on the nucleon formfactors $e, \mu_{p,n}$. Kinematical quantities appearing in the last terms (M_A is the nucleus mass) are defined as

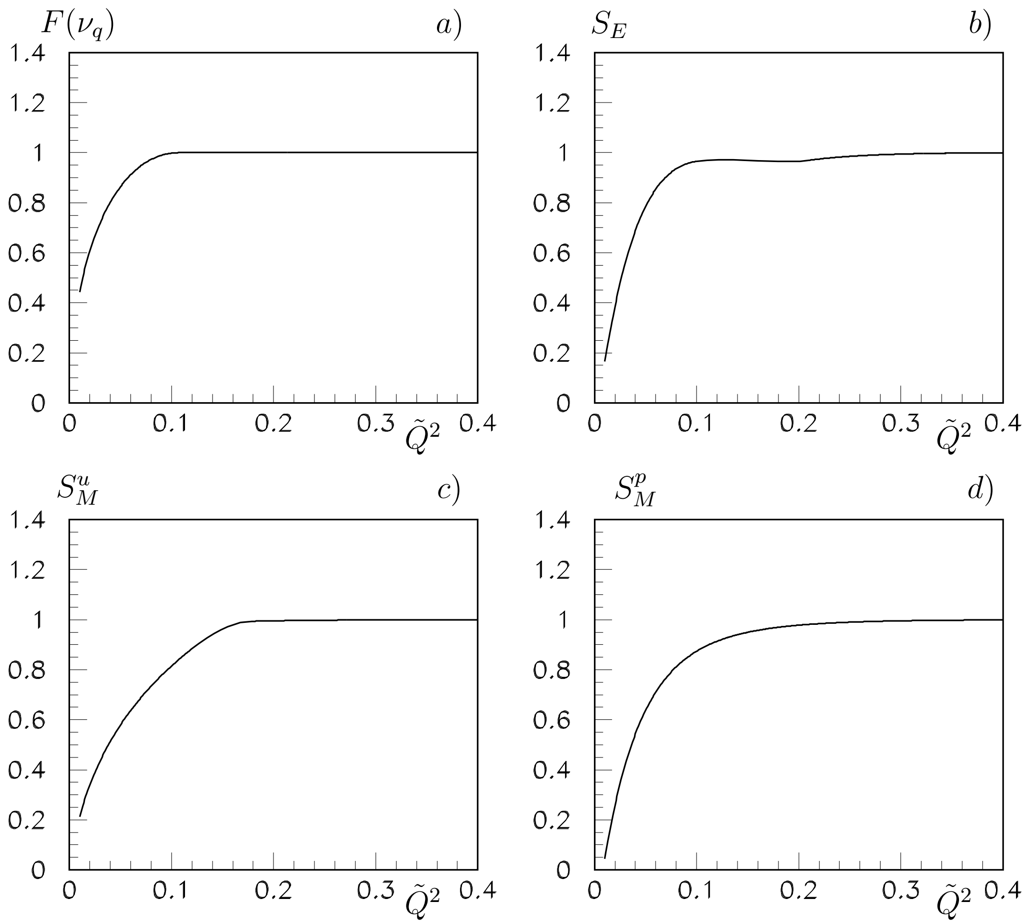

Figure 1. The $\tilde{Q}^2$ dependence of suppression factors calculated within the y-scaling hypothesis (1a) and within the sum rules formalism (1b - 1d).

$$
\begin{aligned}
&f_A = 1 + 2E_1/M_A \sin^2(\bar{\theta}/2), && \eta_A = Q'^2_A/4M_A, \\
&Q'^2_A = 4E_1^2/f_A \sin^2(\bar{\theta}/2), && \rho_A = 1/(1+\eta_A), \\
&\bar{\theta} = 2\sin^{-1}\sqrt{\tilde{Q}^2 M/(2E_1(ME_1 - \tilde{Q}^2))}, && q_a = \sqrt{Q^2_A \rho_A},
\end{aligned}
\qquad (15)
$$

and

$$
v_{TA} = \tfrac{1}{2}\rho_A + \tan^2(\bar{\theta}/2), \quad v'_{TA} = \tan(\bar{\theta}/2)\sqrt{\rho_A + \tan^2(\bar{\theta}/2)}. \qquad (16)
$$

Correspondent quantities without index A are obtained by substitution $M_A \longrightarrow M$.

4. Numerical analysis for HERMES kinematics

In the sections above we derived expressions for the contribution of the QRT to the total correction of DIS on polarized ^{3}He. All quantities used, except for the suppression factors are well defined or have been measured with good accuracy. The suppression factors thus constitute the main uncertainty in the calculation of the QRT. In this

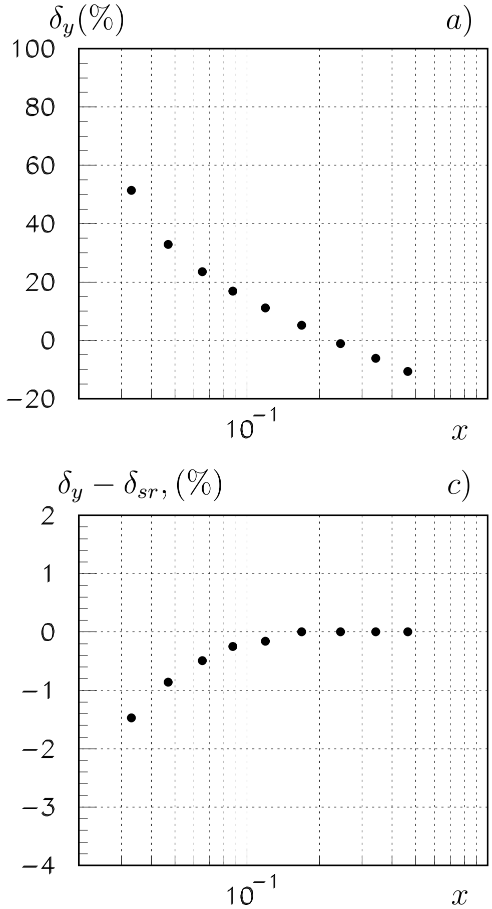

Figure 2. Relative correction to cross section δ calculated within the y-scaling hypothesis (2a), within the sum rules formalism (2b) and the difference (2c). HERMES kinematics (see Table 1 of ref.[15]).

paper we considered two different approaches to the calculation of the suppression factors. The $\tilde{Q}^2$ dependence of suppression factors calculated within the y-scaling hypothesis (fig.1a) and within the sum rules formalism (figs.1b - 1d) is presented in Fig.1. The predicted behaviour for suppression factors is 1 for big values of $\tilde{Q}^2$ and when $\tilde{Q}^2 \to 0$ suppression factors also goes to 0. The bend point correspond to the value $\tilde{Q}^2 = 2k_f \approx 0.1GeV^2$, where k_f is helium-3 Fermi momentum. As can be seen from Fig. 1 the difference in calculation comes from the region below the bend point, but this region is extremely important as due to the behaviour of elastic formfactors with $\tilde{Q}^2$, the region $M^2\frac{x^2}{1-x} \lesssim \tilde{Q}^2 \lesssim 2k_f$ gives the biggest contribution to the calculation of the integral (8).

To investigate the systematic error when measuring the observables it is convenient

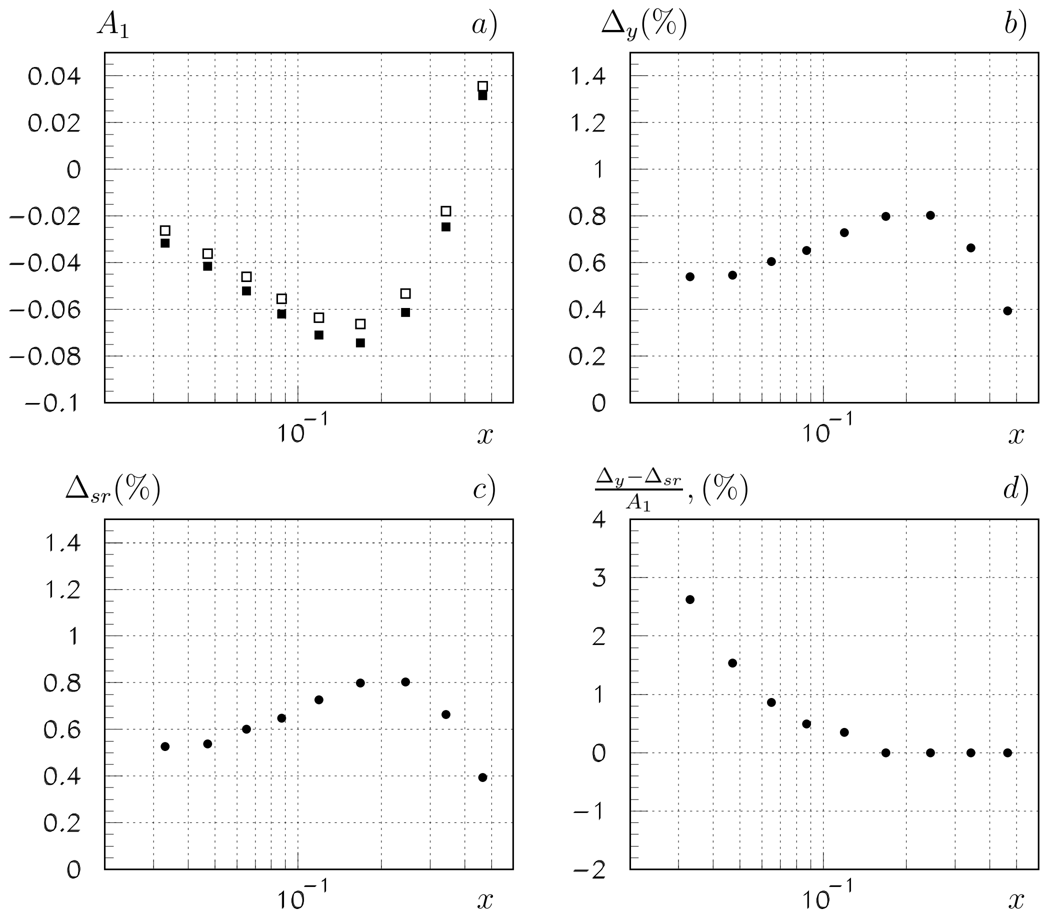

Figure 3. Born (open squares) and observed (full squares) ^{3}He asymmetry (3a). Relative correction to asymmetry Δ calculated within the y-scaling hypothesis (3b), within the sum rules formalism (3c) and the difference (3d) normalized over Born asymmetry. HERMES kinematics (see Table 1 of ref.[15]).

to define quantities δ and Δ:

$$\begin{aligned} \delta_y &= \sigma_y^{obs}/\sigma_0 - 1, \quad \Delta_y = A_{1y}^{obs} - A_1^{born}, \\ \delta_{sr} &= \sigma_{sr}^{obs}/\sigma_0 - 1, \quad \Delta_{sr} = A_{1sr}^{obs} - A_1^{born}. \end{aligned} \tag{17}$$

The indices y and sr correspond to observable quantities calculated within the y-scaling and sum rule approaches. The spin ^{3}He asymmetry is defined by the standard way

$$A_1 = \frac{1}{D}\frac{\sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\uparrow\downarrow} + \sigma^{\uparrow\uparrow}} = \frac{g_1(x,Q^2)}{F_1(x,Q^2)}, \tag{18}$$

and is roughly three times smaller the neutron spin asymmetry measured in [15]. D is the depolarization factor and F_1 and g_1 are ^{3}He structure functions. The results of numerical calculations of the quantities (17) are presented in Figs. 2a-2b and 3b-3c. For spin-dependent structure function the model given in Appendix of ref.[16] is used. The kinematical points (taken from Table 1 of ref.[15]) cover the kinematical region of

the HERMES experiment. Large corrections occur mainly in the first bin due to the large values of y there. Fig. 3a shows results for the asymmetry, with (observed) and without (Born) the total RC.

It should be noted that the results presented in Figs. 2c and 3d give an estimate of the relative systematic uncertainty on ^{3}He spin dependent structure functions due to the radiative tail from the quasielastic peak (see also [17]). It is of the order of a few of percent in the kinematical region of modern polarization experiments.

All numerical data were done for kinematical condition of HERMES, however results for TJNAF are practically the same. The contribution of quasielastic radiative tail is practically defined by y and x and there is no essential dependence on incident electron energy. y defines the probability of the radiative subprocess, but x fixes the low limit of integration over $\tilde{Q}^2$ (or τ). We note also that due to integration main important region is relatively small Q^2. For current experiments in DESY and TJNAF this region is $Q^2 < 1\mathrm{GeV}^2$.

5. Discussion and Conclusion

This paper is devoted to studying of radiative tail from quasielastic peak in deep inelastic scattering on polarized and unpolarized ^{3}He target. Its contribution to total RC is very important but the most insufficiently studied part of standard procedure of radiative correction of experimental data. The main problem is that contrary to the case of elastic formfactors and DIS structure functions the ^{3}He quasielastic response functions are not well studied yet.

In our paper we tried to use some general properties of these functions like y-scaling hypothesis, estimations of sum rules and simply a fact of a presence of a peak at $Q^2 = 2M\nu$ in order to develop and compare different approaches for calculation of the correction. It is clear that the best formula is exact expression (7), but it requires unknown information about responses as functions of two variables ν and Q^2. So it is necessary to look for some compromise between accuracy and uncertainties. We considered two possible approaches. First one is related to y-scaling hypothesis, second one gives an expression in terms of sum rules for electron-nuclei scattering. The results were analyzed and compared numerically. Difference between them gives an estimation on systematical uncertainty due to quasielatic radiative tail. We understand that consideration and comparison of only two approaches is not enough to make a conclusion about systematical error. Unfortunately in literature there are no known to us possibilities to construct any other approach which can provide explicit information for some necessary quantities in $\vec{\mathrm{e}}^3\vec{\mathrm{He}}$ inclusive quasielastic scattering. If such calculations or data will appear the generalization of the developed approach is straightforward.

For the current situation we recommend to use formulae (8) and (9) as basic to calculate radiative tail from quasielastic peak. In our opinion the effect of the assumptions on these formulae is minimum. Furthermore these assumptions can be controlled better and cannot lead to cruial results due to unknown ingredients. But

further progress have to be connected with exact formula (7). For that new experimental data or/and new reliable models have to appear.

Acknowledgments

The work of IA was partially supported by the U.S. Department of Energy under contract DE–AC05–84ER40150.

References

[1] S.Stein et al., Pys.Rev. **D12**(1975)1884
[2] J.Bailey et al., Nucl.Phys. **B151**(1979)367
[3] L.Mo, Y.Tsai, Rev.Mod.Phys. **41**(1969)205;
Y.Tsai, SLAC-PUB-848, (1971).
[4] I.V.Akushevich, N.M.Shumeiko, J.Phys., G **20**(1994)513
[5] I.V.Akushevich, A.N.Ilyichev and N.M.Shumeiko, J.Phys. G **24**(1998)1995
[6] P.Hoodbhoy, R.L.Jaffe and A.Manohar, Nucl.Phys. B**312**(1989)571
[7] I.Akushevich et al., Comp. Phys. Comm., **104** (1997)201
[8] W.Leidemann, E.Lipparini and S.Stringari, Phys.Rev. **C42**(1990)416
[9] G.B.West, Phys.Rep. **18**(1975)263
[10] A.K.Thompson et al., Phys.Rev.Lett. **68**(1992)2901
[11] T. de Forest and J.D.Walecka, Adv.Phys. **15**(1966)1
[12] E.J.Moniz Phys.Rev. **184**(1969)1154
[13] G.Orlandini and M.Traini, Rep.Prog.Phys. **54**(1991)257
[14] R.Schiavilla, D.S.Lewart, V.R.Pandharipande, S.C.Pieper, R.B.Wiringa, and S.Fantoni Nucl. Phys. **A473**(1987)267
[15] K.Ackerstaff et al., Phys. Lett. B**404**(1997)383
[16] N. Gagunashvili et al., Nucl. Instrum. Meth. **A412** (1998) 146.
[17] I. Akushevich and A. Nagaitsev, J. Phys. **G24** (1998) 2235